Cracks of fundamental quantum physics

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Abstract

The fundaments of quantum physics are still not well established. This paper tries to find the cracks in these fundaments and suggests repair procedures. This leads to unconventional solutions and a new model of physics. As part of this enterprise an underpinning of the existence of strands is provided. Another innovation is the derivation of a curvature field from the superposition of all other fields. The most revolutionary introduction is the representation of dynamics by a sequence of separable Hilbert spaces. Together, this embodies a repair of fundaments that does not affect the building.

History

In its first years the development of quantum physics occurred violently [1]. As a consequence some cracks sneaked into the fundaments of this branch of physics. A careful investigation brings these cracks to the foreground. The endeavor to repair these cracks delivers remarkable results.

In the early days of quantum physics much attention was given to equations of motion that were corrections of classical equations of motion. The Schrödinger approach was one and the Heisenberg approach was another. Schrödinger used a picture in which the state of a particle changes with time. The operators that act on these states are static. Heisenberg uses a picture in which the operators change with time, but the states are static. For the observables this difference in approach has no consequences. This fact is important. It shows that time is just a parameter instead that it acts as a property of physical items[1]. Later Garret Birkhoff, John von Neumann and Constantin Piron found a more solid foundation that was based on quantum logic. They showed that the set of propositions of this logic is isomorphic with the set of closed subspaces of an infinite dimensional separable Hilbert space, whose inner product is defined with the numbers taken from a division ring. The ring can be the real numbers, the complex numbers or the quaternions. Since then many physicists do their quantum physics in the realm of a Hilbert space. However, the Hilbert space has no operator that delivers eigenvalues for parameter time.

Cracks in the fundaments

Fist scratches

These physicists quickly encountered the obstinate character of the separable Hilbert space. Its normal operators have countable eigenspaces. This can still correspond to a dense coverage of the corresponding hyper complex number space. However, this space is no continuum. Thus, functions defined using these eigenspaces as parameter domains cannot be differentiated. In order to cope with this defect, most physicists resorted to other types of Hilbert spaces than the separable Hilbert space, but in doing so they neglect that in this way the stringent relation with quantum logic gets lost.

Severe defects

Further, it appears that the separable Hilbert space cannot represent physical fields and cannot represent dynamics. This is a severe drawback and it looks as if the switch to the other Hilbert spaces becomes mandatory. For example quantum field theory represents fields as operators that reside in a non-separable Hilbert space. In this paper the strategy is to hold strictly to the link with traditional quantum logic. So the road that is taken by quantum field theory is not followed.

[1] Later this fact is used in order to apply the progression step counter as a parameter that characterizes the members of a sequence of Hilbert spaces.

Back to the future

On the other hand there are more and more signals that nature is fundamentally granular and the non-separable Hilbert spaces do not provide that feature. This guides backwards to the separable Hilbert space. But in that case we must learn to live with this granularity. In addition we must find other ways to represent fields.

Dynamic way out

The non-separable Hilbert spaces including the rigged Hilbert space gave similar problems with representing dynamics as the separable Hilbert space does. There is no place for **time** as an eigenvalue of an operator neither in separable Hilbert space nor in the other Hilbert spaces. For that reason, it is better to accept that the separable Hilbert space can only represent a static status quo.

Granularity

Nature is fundamentally granular. The so called Planck units [2] are designed using dimension analyses, but it is generally accepted that below these limits (Planck-length, Planck-time, Planck constant = unit of action and Boltzmann's constant = unit of entropy) no discerning observation is possible. One could say that below these limits nature does not exist or that nature just steps over these regions. The Planck-length and Planck-time are related to the Planck constant, the speed of light c and the gravitational constant G. It is not said that nature's granularities have exactly these sizes. The Planck units are derived by dimensional analysis. The Planck unit sizes rather form an order of magnitude indication, but these measures are useful and we do not have a better estimate. The mutual relation between these units is important. For example, the ratio between the Planck-length and the Planck-time equals the speed of light c. If you reckon that at every time step a physical item can at the utmost take one space step, then the maximum speed of all physical items is automatically set at the speed c.

Coping with granularity

A solution must be found for the fact that GPS-like normal operators in separable Hilbert space possess granular eigenspaces. A GPS operator would have a lattice-like

[2] http://en.wikipedia.org/wiki/Planck_units

eigenspace of densely packed granules. The lattice would possess preferred directions. This does not correspond with physical reality. It means that it is impossible to define an operator that acts like a proper global positioning system (GPS), which is required in the positioning of field values or when we want to relate Hilbert vectors with position.

The separable Hilbert space can provide a GPS-like operator that offers a dense coordinate system as its eigenspace. An eigenspace consisting from all rational quaternionic numbers would be countable and thus it can be an eigenspace of a normal operator in separable Hilbert space.

However this eigenspace is no continuum and as a consequence it does not support differentiation.

Further, a GPS-like operator is not granular. In nature the space of the positions is granular and the size of the granules is of the order of the Planck-length, which is $1.6 \cdot 10^{-35}$ m. The lattice formed by the densely packed granules in the eigenspace of a granular GPS-like operator would immediately show unnatural preferred directions. Such situations can occur in condensed matter, but that is an exceptional condition.

A dense packaging of granules may occur in horizons. For example, horizons of black holes appear to be covered by a dense package of granules.

Apart from these exceptions the exclusion holds for any multidimensional subset of eigenvalues, even if it contains a countable number of values that are taken from a continuum.

The required granularity is special. It is a granularity of differences rather than a granularity of values. This might guide the way to a solution.

Background coordinates

The separable Hilbert space is connected with its Gelfand triple, which is also called a rigged Hilbert space. In fact the rigged Hilbert space is only called that way, because it is no Hilbert space. A background coordinate system exists in rigged Hilbert space as the eigenspace of a GPS-like operator, but when we insist on granularity of the GPS-like coordinates, it cannot be directly used in separable Hilbert space in order to locate Hilbert vectors in a regular way. As signaled before, this will introduce anomalies. So, we must find an indirect way. This is delivered by the strand operator[3], which resides in separable Hilbert space and has an equivalent in the rigged Hilbert space. There it can be **coupled** to the background coordinate system. Apart from horizons, the eigenspace of the strand operator does not contain multidimensional sets of eigenvalues. Instead, it contains chains of granules. Thus, in separable Hilbert space it avoids the mentioned problems.

Strand operator

The mentioned coupling between the eigenspace of the strand operator in separable Hilbert space and the eigenspace of the GPS-like operator in the rigged Hilbert space is not precise. That inaccuracy causes that the eigenvalues of the strand operator have a stochastic nature. Their spread has a minimal value. So the eigenvalues can be seen as granules.

When the past and the future of the eigenvalues are kept in sight, then the eigenspace of the strand operator contains a set of chains that are put together from granules. In the chain the granules are ordered. In each chain one granule is exceptional. We call it the **current granule**. The part of the chain that ends just before the current granule is called the past sub-chain. The part that starts just after the current granule is the future sub-chain.

One could ask whether having only the current granule could be sufficient. For the model, the direct neighborhood of the current granule is the most relevant part of the chain. The rest of the chain is hardly used. It only gives a reflection of a possible past and a possible future which is derived from the current field configuration. However,

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[3] The name strand operator is related to the strand model of Christoph Schiller that brought me the idea to use chains as a solution.

the step to the next version of the “current granule” is taken inside the chain. **At a given progression step maximally one space step is allowed.** When that step is taken, then on the average that step has the size of one granule.

The step to the next granule is controlled by a probability density distribution (PDD). The extent of this PDD is set by the properties of the stochastic coupling between the background coordinate system and the position of the granule. **In its minimal format** the stochastic coupling has characteristics that are similar to the characteristics of the ground state of a quantum harmonic oscillator. This minimal extent is of the order of the Planck-length. This is why the granules appear to have a basic size of the order of the Planck-length and seem to be surrounded by a PAD that can take a wider extent. The quantum harmonic oscillator is only mentioned as an example. The actual form of the wider extent of the PAD may be quite different. It depends on the characteristics of the particle that makes use of this granule as its anchor point.

At each actual step a space analog to the space covered by the ground state is inaccessible. Nature steps over this space and lands in the middle of a new current granule.

One might ask why this restriction exists. The reason must be sought in the combination of **stochastic inaccuracy** with the **atomicity** of quantum logic. This restriction goes further than countability.

Chains can split and they can merge. The corresponding creation and annihilation occurs during a progression step and is controlled by PAD’s that are attached to the current granules.

The chains in the eigenspace of the strand operator are causal chains.

Statistics

The PAD is a constituent of the field that surrounds the granule. The creation and annihilation operators of fields have eigenfunctions that are Poisson distributions. Such distributions are produced by Poisson processes. A Poisson process can be combined with a subsequent binomial process in order to form a generalized Poisson process that has a lower efficiency than the original Poisson process. The efficiency is weakened by the weakening that is introduced by the binomial process. The spatial spread introduced by the PAD can be interpreted as a binomial process with a spatially varying weakening factor. The spread function is equal to the squared modulus of the PAD.

Canonical conjugate

Depending on the type of the particle that anchors on the granule there may be many types of PAD's. Near the anchor point the basic shape of the PAD's are all equal. Apart from a factor (1, ***i***, -1 or –***i***) they are invariant under Fourier transformation. This means that near the anchor point the eigenspace of the canonical conjugate of the strand operator has the same basic format as the eigenspace of strand operator. It also anchors on similar granules.

Strand space

The strand operator has an outer horizon. Outside this horizon its eigenspace does not contain granules. It might also have inner horizons such that inside these inner horizons no granules exist.

Most inner horizons are borders of black holes. These horizons are bubbles that consist of densely packed granules. The PAD's that are attached to these granules have taken their minimal possible size. Each granule is connected to a Hilbert vector which is eigenvector of the strand operator. That Hilbert vector represents a quantum logical proposition. It carries a single bit of information that indicates its membership of the eigenspace of the strand operator. The inner horizons form an exception to the rule that the granules must not form a multidimensional subset.

Other horizons

Since light transports all information and has a limited speed, the eigenspace of the strand operator may feature information horizons. Every object in space has its own private information horizon. This horizon is in fact the image of a start horizon that occurred at the start of the universe. The start horizon is a special kind of inner horizon that was at the same time an outer horizon. It can be interpreted as a bubble that existed in empty space and that suddenly converted into matter[4]. From that moment the granules that formed this special horizon spread over space and their PAD folds out, such that it takes more space than just the size of the granule. This occurrence must be unique or its probability must be very low. There is no indication that it happened more often during the lifetime of the universe.

Affine space

Since the unit sphere of the separable Hilbert space is an affine space and all eigenvectors of the strand operator are represented in that space, the strand operator can be considered to have no origin or the origin is just arbitrarily selected. The same consideration holds for the GPS-like operator in the rigged Hilbert space.

Types of chains

The chains may be closed or they start and end at a horizon. Further they may split and merge. This corresponds with creation and annihilation of particles that anchor on these chains. Actually, only the direct environment of the current granule of the chain is relevant. The granules in short closed chains may represent the anchors of virtual particles. These granules are virtual granules.

The generation and annihilation of particles occurs for example in field configurations that are locally invariant under Fourier transformation, such as linear and spherical harmonics.

The chains have much in common with the strands in Schiller's strand model. However, they are not exactly the same.

Vacuum

The inaccuracy in the coupling between the background coordinate system and the granules also plays a role in the space where no current granules exist. In this space

[4] See Birth of the universe

virtual granules may exist during a very short period, for example during a single progression step. These virtual granules form the content of vacuum.

Virtual granules only occur inside the outer horizon and outside the inner horizons of the strand operator. The virtual granules can be interpreted as the ground state of harmonic oscillators. This ground state corresponds with the minimal extend that the PAD can take. The vacuum has a constant density $\rho_{vac}$of virtual granules.

In the Hilbert book model the space between horizons is stochastically, but on the average uniformly covered with virtual granules. At every progression step these virtual granules are redistributed. The actual granules exist in between these virtual granules, but they possess a wider spread of the corresponding PAD's. These wider PAD's tend to last longer at (nearly) the same location.

Fields

Fields do not fit inside a separable Hilbert space. Any field would cover the whole Hilbert space. Every Hilbert vector would touch a value of the field. Which value is touched, depends on the functionality of this vector. When the vector is one of the eigenvectors of a normal operator and when the field can be expressed as a function of the eigenvalues of this operator taken as the parameter of the function, then the field value would correspond with the parameter value that equals the eigenvalue that corresponds to eigenvector. In that case, the considered field value will be connected to the considered vector.

In superposition, field values may compensate each other. That is possible when they have opposite sign.

Function of the field

The function of the physical fields is to take care of minimizing changes during dynamical steps. This function becomes evident when dynamics is implemented. Fields keep the shape of the chains of the strand operator smooth. In first instance the private fields influence the chain at their anchor point. Due to their extent, the fields also influence other chains.

Basic field constituent

A PAD that is attached to the current granule takes care of the fact that the chain in the neighborhood of the current granule stays sufficiently smooth. This becomes important when dynamics is implemented because with each dynamic step the current granule either stays at its current position or it moves one place ahead in the chain.

It must be noticed that exactly this restriction is the reason why speed has a maximum! The ratio of the space step and the time step equals the speed of light.

The squared modulus of the PAD is a probability density distribution (PDD). It determines the probability of the position of the current granule. The probability is large when the position is close to the position of the previous current granule.

Fields influence the chain

The private fields overlap and because they are all PAD's their superposition causes an interaction between the particles that anchor on these fields.

Taken over a sequence of dynamic steps, the chain appears to fluctuate. The fluctuation determines the probability distribution and vice versa the dynamic changes of the probability density distribution determine the fluctuations of the chains. This relation is instantaneous. There is no causal relation. (The granules are ground states of field constituents).

If the chains would be observable, then the probability distribution could be determined by averaging the fluctuations over some period. However, neither the chains, nor the probability amplitude function are directly observable items. Only their effects become observable.

Particles

The Hilbert book model leaves open whether depending on its type, an elementary particle relates to one or more of these chains. In any way the current granules of these chains are related to the current section of the path of the particle.

Strands, curvature, torsion and chirality

The idea to attach more than one strand to a particle is taken from **Christoph Schiller's strand model**[2].

In contrast to torsion, curvature relates to mass. For example, according to Schiller's strand model, the strand that represents a massless photon has a helix shape. The strands that represent the massive W bosons have the shape of an overhand knot. Since this knot shows chirality, it possesses electric charge. The strands that represent the massive Z bosons have the shape of a figure eight knot. Because the figure eight knot features no handedness, it does not possess electric charge. In a similar way Christoph Schiller attributes properties to all elementary particles.

The Hilbert book model does not use the strand concept of Schiller's strand model. Strands and chains are both one dimensional and both interact with fields, but that is how far the resemblance goes.

Extended Hilbert space

The addition of PAD's to the Hilbert vectors that are attached to the current granules extends the separable Hilbert space to a new construct. For that reason we call this new construct an **extended separable Hilbert space**.

Extended quantum logic

Via the relation between the separable Hilbert space and traditional quantum logic we can extend quantum logic to an extended quantum logic that includes physical fields in a similar way as the extended separable Hilbert space model does. It means that a subset of the propositions is afflicted with a stochastic inaccuracy that can be characterized by a probability distribution.

Covering field

The PAD that is connected to the current granule is a basic field constituent. The superposition of all these basic constituents forms a covering field. The configuration of the covering field depends on the configuration of the elementary particles. When the configuration of chains changes, then the configuration of particles changes and the covering field changes accordingly.

Curvature field

According to Helmholtz decomposition theorem, the static version of the covering field decomposes into a rotation free part and one or two divergence free parts. The local decomposition depends on the local field configuration and in general it does not run along straight coordinate lines. The local decomposition into a one dimensional longitudinal part and a transverse part defines a local curvature. This curvature can be used to define a local metric. This metric is a tensor and on its turn it can be used to define a derived tensor field. We will call this the curvature field. It has all aspects of the gravitation field. When split back into curvature fields that are private to the particles the private curvature field can be used to attach the property "mass" to the corresponding particle.

What is curvature?

In order to comprehend quantum physics, it is sometimes sensible to step one dimension down. Optics is in many respects similar in 2D to quantum physics in 3D.

When optics is studied, then it is often done by following the live path of a point object. This can be done by ray tracing and it can be done by applying Fourier optics. When the quality of imaging equipment must be specified in an objective way, then it is often done in terms of the Optical Transfer Function[5] (OTF). The OTF is defined as the Fourier transform of the 2D spatial spread of the point object. This definition supposes the presence of a projection surface. In practice the analyzed area is kept rather small. Further the energy contained in the point image is insufficient to activate the measuring equipment. For that reason the measurement is done by analyzing the image of a short thin slit object. Provided that the point image is spatially invariant in the area of the slit object, the analyzed image is the convolution of the Point Spread Function[6] (PSF) and the slit object. After taking the Fourier transform the analyzed image is the product of the OTF and the Fourier transform of the slit object. This last function is a two dimensional sinc function that extends in the direction across the slit in which the slit is small and is thin in the direction in the direction along the slit. The result corresponds closely to a vertical 1D cut through the OTF. When the PSF is rotationally symmetric, then the result is independent of the direction of the slit. The Modulation Transfer Function is the modulus of the MTF. Any vertical cut through the MTF is symmetric. Thus when the PSF is not rotationally symmetric usually two measurements of the MTF

[5] http://en.wikipedia.org/wiki/Optical_transfer_function
[6] http://en.wikipedia.org/wiki/Point_spread_function

are specified. The first result is the one with maximal extent and the other has minimal extent.

When the imaging system is a rotationally symmetric lens, then on-axis the PSF is rotationally symmetric, but off-axis the Seidel aberrations take their toll and the PSF is no longer rotationally symmetric. In that case a radial (longitudinal) OTF and a lateral (transverse) OTF are specified.

We only traced one ray. Actual images are constituted of the combined PSF's of an extended object. In this way the PSF is a constituent of a scalar field. **The divergence and the curl of that scalar field form a vector field**. According to Helmholtz theorem the vector field can be split in a rotation free component and a divergence free component. In the above situation these components are the longitudinal and the transverse components.

Now exchange the lens against an arbitrary but smoothly shaped glass body. The direction of the longitudinal component no longer runs along a straight line. The curvature of the decomposition defines a local curvature. This 2D situation looks more like the situation that we have in 3D quantum physics.

In short: In optics the actual field configuration corresponds to a curvature of the coordinate system in which the PSF is spatially invariant.

About the field concept

It is common practice to treat the EM fields and the gravitation field as different and independent subjects. In this interpretation, the gravitation field generates the curvature of the coordinate system in which the other fields must operate.

The Hilbert book model takes a different approach. It puts the reason for the curvature of the coordinates in the properties and configuration of the covering field. The curvature that exists in this way is used to derive the curvature field. On its turn the curvature field determines the values and locations of actual or virtual masses. The wave function is also interpreted as a constituent of the covering field. In this way it also contributes to the curvature field. This picture unifies all fields.

The PAD's can be seen as a reflection of the stochastic inaccuracy of the coupling between the eigenspace of strand operator and the eigenspace of the GPS-like operator that resides in rigged Hilbert space and acts as background coordinate system. In the same way the curvature field can be seen as an administrator of the deficiency of this coupling as is marked by the local curvature.

The start horizon

With this concept of the curvature field the field configuration near the origin of the expanding universe can be interpreted as to be generated completely by the curvature that corresponds with the local geometry. This curvature determines the field values of the local curvature field. This curvature field corresponds to a virtual mass that represents the influence of that local geometry. This virtual mass does not correspond to the presence of actual matter. It just represents the particular geometry that is present near the origin of the universe.

In the Hilbert book model the universe starts with a bubble shaped horizon, which is at the same time an inner horizon and an outer horizon. This start horizon[7] consists completely out of densely packed granules. At the start the size of these granules is quite large. These granules do not represent ground states of a corresponding PAD. They represent a much higher state. Like the ground state this state offers the capability to form bubbles. The start horizon is instable. Its granules collapse into a new format whose size is many orders of magnitude smaller. As a consequence the space that was taken by the start horizon gets filled with a diffused set of the smaller granules that can move around freely. As a consequence, part of these granules recombines into new smaller bubbles. These smaller bubbles are black holes. These new inner horizons contain a lower amount of granules and the granules are much smaller than in the start horizon. They represent the ground state of the PAD. Other granules form loosely connected assemblies. Still others keep moving free. For the free and loosely packed

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[7] The idea of the existence of a start horizon is a speculation. Inner and outer horizons exist.

granules the PAD’s unfold. This unfolding results in a large multitude of different private field types.

The result of this procedure is that the original geometry converts partly into matter. Some of it converted back into geometry. This happening is then the start of a new expanding universe. However, after this first implosion the expansion can be described as a metric expansion.

This also indicates what happens when a large mass collapses into a black hole. The matter disappears and converts into a strongly curved geometry.

The most important message is that the geometry determines the curvature field, rather than that the curvature field determines the geometry. This can go so far that the geometry not only determines a virtual mass, but under the proper circumstance it can also generate actual matter that corresponds to the virtual mass. What happens during the collapse of a large mass into a black hole is not only the generation of the horizon. It is also the folding together of the private fields that existed in the surround of the anchor points until they reach their smallest possible extent.

What also becomes clear is that the configuration of the anchor points in combination with the type of the private fields determines the curvature of the geometry.

In this picture the gravitation field only acts as an administrator. The real actors are the Hilbert vectors that correspond to the anchor points and the corresponding private fields.

Canonical coordinates

We start with the situation in which we can select ideal coordinates. What that means will become clear soon.

Ideal coordinates

The inner product of the Hilbert space can be used to relate two orthogonal bases that are each-other's canonical conjugate. In a quaternionic Hilbert space this is not a straightforward procedure. Luckier wise, the quaternionic number space can be divided into a series of complex number spaces. We just chose one imaginary direction and do as if we are in complex Hilbert space. However, this singles out that particular direction. We may choose the direction in which the local longitudinal direction of the covering field runs. The definition of longitudinal is in fact taken in the canonical coordinate space of the current configuration space. It can be any radial direction taken from the origin of that space. This may give problems when the original configuration space is curved, thus when the longitudinal direction changes with location.

The fact that space is curved follows from the fact that when this space is covered with shapes that should all have the same form; the form of the shape in fact changes with the location of that shape[8].

For the moment we assume that we have selected a coordinate system for which the selected longitudinal field direction runs along a straight line and stays that way. We do not bother about granularity, because we will base our investigations on fields that are specified using a continuum background Global Positioning System coordinates. In Fourier spaces we need the corresponding Global Momentum System coordinates. So we pick the eigenspace of a normal GPS-like operator $\breve{\mathrm{Q}}$ that resides in rigged Hilbert space as our coordinate system. It has an equivalent GPS-like coordinate operator Q in separable Hilbert space whose eigenspace lays dense in the eigenspace of the rigged Hilbert space GPS. The operator $\breve{\mathrm{Q}}$ is selected such that the selected longitudinal direction of the field runs along one of the imaginary base vectors of the eigenspace. The set of eigenvectors $\{|\mathrm{q}>\}$ of operator Q forms an inner product with another

[8] See: What is curvature.

normal operator P which is the canonical conjugate of Q. The eigenvector $|\mathrm{q}>$ corresponds to an eigenvalue q and similarly the eigenvector $|\mathrm{p}>$ of P corresponds to an eigenvalue p. The inner products are now given by:

$$< \mathrm{p}|\mathrm{q} > = \exp(\mathbf{i}\hbar\mathrm{qp}) \qquad (1)$$

The constant h in $\hbar = 2\pi\mathrm{h}$ is Planck's constant. The imaginary 3D base vector **i** of the quaternionic number space is the imaginary base number of the selected complex number space.

This procedure can be performed for the two operators and three mutually perpendicular imaginary base vectors of the eigenspace. We have defined the procedure for the operators P and Q that reside in separable Hilbert space, but with respect to its application to Fourier transforms, it makes also sense for the equivalent operators $\breve{\mathrm{P}}$ and $\breve{\mathrm{Q}}$ in rigged Hilbert space. Their eigenspaces form a continuum.

Fourier transform

It can easily be seen that the specified inner product also defines a complex Fourier transform. We start with the separable Hilbert space. By taking all three dimensions the specified inner product defines the imaginary part of a quaternionic Fourier transform.

$$< \mathrm{q}|\mathrm{f} > = < \mathrm{f}|\mathrm{q} >^* = \mathrm{f}^*(\mathrm{q}) = \sum_{\mathrm{p}} (< \mathrm{q}|\mathrm{p} > \cdot < \mathrm{p}|\mathrm{f} >) \qquad (1)$$

And reversely:

$$< \mathrm{p}|\mathrm{f} > = \tilde{f}(p) = \sum_{\mathrm{q}} (< \mathrm{p}|\mathrm{q} > \cdot < \mathrm{q}|\mathrm{f} >) \qquad (2)$$

It must be reckoned that these are discrete transforms. Here the **Hilbert function**

$$f(q) = < f|q > \qquad (3)$$

is a sampled function and is transformed in formula (2) into its Fourier partner $\tilde{f}(p)$.

In rigged Hilbert space the sum becomes an integral.

Use of the Fourier transform

In separable Hilbert space, Hilbert functions are sampled functions and are constructed from the eigenvectors and eigenvalues of a normal operator and a selected Hilbert vector. See formula (3).

The discrete transform and the Hilbert functions do not have many usages. In practice the Fourier transform is applied to Hilbert fields[9] rather than to Hilbert functions.

The Fourier transform of a quaternionic field must be performed with a quaternionic Fourier transform that acts in a continuous number space [3].

The Fourier transformation of a private field[10] of a particle does two things. It shifts from a GPS-like coordinate system to its canonical conjugate GMS-like coordinate system. Apart from that it transforms the private field from a quantum cloud into a wave package. This new probability distribution tells about momentum rather than about position.

Fourier transform habits

A Fourier transform keeps inner products invariant. Thus it is a unitary transformation. It has no eigenvectors and as a consequence it has no eigenvalues. However, in rigged Hilbert space functions exist that apart from a multiplication factor are invariant under

[9] Distributions and fields: Hilbert fields
[10] Distributions and fields: Hilbert fields: Private field

Fourier transformation. Examples of these are the functions that describe linear and spherical quantum harmonic oscillators. The multiplication factor can be 1, i, -1, or -i.

In separable Hilbert space, the Fourier transform converts an orthogonal base into another orthogonal base, which is completely distinct from the original base. Any member of the second base is a linear combination of all members of the first base. The modulus of all coefficients in this linear combination is equal to unity. In rigged Hilbert space the function $\exp{(i\, p\, q\, \hbar)}$ and the Dirac delta function $\delta(q)$ form Fourier transform pairs. In separable Hilbert space the Kronecker delta replaces Dirac's delta function.

The existence of canonical conjugation is the reason of the weakening of the modular law that makes the difference between classical logic and quantum logic.

A very important property of Fourier transforms is that it transforms a differentiation into a multiplication with the canonical conjugated coordinate. This only works in rigged Hilbert space. In the Hilbert book model it is applied to Hilbert fields[11].

Actual coordinates

In practice the ideal conditions are seldom valid and if they are valid, they are only valid locally and with reduced accuracy. It means that the inner product that defines the canonical conjugate has only local validity and the same holds for the Fourier transforms that are specified with the aid of that inner product.

In actual situations depending on the field coordinates the coordinate system gets curved locally. Only an appropriate coordinate transformation can bring us back to the ideal situation. This is a purely mathematical activity and the required transform changes with the field configuration that resides in the current static status quo. It does not affect physical reality. So if we know how to perform this coordinate transformation then physics in this static status quo becomes trivial. This is the reason why particles move along geodesics. However, in another static status quo the field configuration will be different. This requires a separate coordinate transformation for every static status quo. The alternative is that we accept a curved coordinate system.

[11] Distributions and fields: Hilbert fields

The presented picture supposes that nowhere the field excitations are so violently that it becomes impossible to define a local curvature.

Coherent state

A **coherent state** is a specific kind of state of the quantum harmonic oscillator whose dynamics most closely resemble the oscillating behavior of a classical harmonic oscillator system.

The coherent state |α> is defined to be the 'right' eigenstate of the annihilation operator $\mathcal{A}$. Formally, this reads:

$$|\mathcal{A}\,\alpha> = \alpha|\alpha> \tag{1}$$

Since $\mathcal{A}$ is not Hermitian, α is a hyper complex number that is not necessarily real, and can be represented as

$$\alpha = |\alpha|\,\exp(k\,\theta) \tag{2}$$

where

θ is a real number.

|α| is the amplitude and

θ is the phase of state |α>.

This formula means that a coherent state is left unchanged by the annihilation or the creation of a particle. The eigenstate of the annihilation operator has a Poissonian[12] number distribution. A Poisson distribution is a necessary and sufficient condition that all annihilations are statistically independent. (Shot noise is characterized by a Poisson distribution. See information detection.)

The coherent state's location in the complex plane (phase space[13]) is centered at the position and momentum of a classical oscillator of the same phase θ and amplitude. As the phase increases the coherent state circles the origin and the corresponding disk neither distorts nor spreads. The disc represents Heisenberg's uncertainty. This is the most similar a quantum state can be to a single point in phase space.

Distributions and fields

The concepts that have been introduced so far invite the introduction of Hilbert distributions and Hilbert fields.

Hilbert distributions

Hilbert distributions are sets of Hilbert vectors, in which each vector corresponds to the current granule of a member of a set of chains. Thus, these vectors are eigenvectors of the strand operator in the current Hilbert space. All past and future granules in a chain correspond with a Hilbert vector in their corresponding Hilbert spaces, but the vectors of a Hilbert distribution correspond with the corresponding current granule, thus with a Hilbert vector in the current Hilbert space.

Also the granules that compose a horizon are eigenvectors of the strand operator. An **elementary Hilbert distribution** is a set of Hilbert vectors that belong to an elementary particle.

Hilbert field

A Hilbert field is a superposition of the PAD's that are attached to the Hilbert vectors in a Hilbert distribution. In principle all Hilbert distributions are Hilbert fields.

[12] http://en.wikipedia.org/wiki/Poissonian
[13] http://en.wikipedia.org/wiki/Phase_space

A **private Hilbert field** is a Hilbert field that belongs to an elementary Hilbert distribution. However, if a complicated particle consists of a set of elementary particles, then we consider the superposition of the private fields of the elementary particles as the private field of the complicated particle. The Hilbert field is a skew field. The Hilbert book model only considers Hilbert fields whose values are taken from a division ring.

The covering field is the superposition of all private fields. It is a Hilbert field

Optics and quantum physics

If all PAD's would be similar, then the Hilbert field can be considered as the convolution of this PAD and a distribution of Dirac delta functions that correspond to the Hilbert distribution. This picture resembles (ideal) ray optics and if we take the Fourier transform then it resembles (ideal) Fourier optics. This is the reason that wave mechanics has so much similarity with optics. The characterization "ideal" indicates the restriction that all blurs are equal. In practical optics the blurs are not equal and change with position in the image surface. In quantum physics the same happens[14], but the blur may also change with the type and properties of the particle.

The Optical Transfer Function characterizes the information transfer capability of an imaging system. In the image plane this OTF has only a local validity and it changes with the angular and chromatic characteristics of the light beam. Also the phase homogeneity of the light plays an important role. In similar way the Fourier transform of the PAD characterizes the information transfer capability of a physical system.

Nothing is said yet about detecting the information that is carried by the particles. That will be treated later[15].

Dynamics

The extended separable Hilbert space model can only represent a **static status quo**. By using this ingredient, dynamics can be implemented by a model that consists of an ordered sequence of such extended Hilbert spaces. It corresponds to an equivalent sequence of extended quantum logics.

[14] See: What is curvature
[15] Information detection

In order to give this model a name, we can call it the **Hilbert book model**. Passing through the sequence is like glancing through a book, where each page describes a static status quo.

The chains of the strand operator pass through a range of Hilbert book pages. A loop must be interpreted as a pair of chains that split at the start and merge at the end. The split and the merge occur between pages.

What is important is that each static status quo holds both the current state and ALL preconditions for the next static status quo. Thus in principle the duration of the progression between subsequent static status quos is unimportant. The Hilbert book model takes all progression steps to be of equal length.

Spacetime

This procedure introduces a new parameter that acts as a global progression step counter. This parameter must not be confused with our common notion of time, which only has local validity.

The dynamic model implies that space is not the only granular quantity. It also means that progression occurs in discrete steps. Further, it indicates that against general acceptance, fundamentally, space and progression have little to do with each other. With other words, no support exists for a fundamental physical spacetime quantity.

That does not say that no relation between the fundamental space step and the fundamental time step can exist. The Minkowski signature is a clear prove of such relation. It can already be understood from the ratio between the Planck-length and the Planck-time. A further more complex relation is set by the properties of space and the properties of the displacement group.

When the smallest possible space step

$$l_{Pl} = \sqrt{\hbar G/c^3} \tag{1}$$

and the smallest possible coordinate time step

$$t_{Pl} = \sqrt{\hbar G/c^5} \tag{2}$$

are put into the Minkowski signature,

$$\Delta\tau^2 = \Delta t^2 - \Delta q^2/c^2 \tag{3}$$

then the corresponding proper time step $\Delta\tau$ is zero.

The number of Planck-time steps equals the number of global progression steps. The number of Planck-length steps must always be equal to or lower than the number of Planck-time steps. A free photon never takes a non-zero Δt step. The number of its space steps always equals the number of its time steps.

Any particle that does not travel with light speed skips some of its space steps. Any particle can take a space step in a direction that differs from the direction of a previous step.

Relativity

Wiki states: "One Planck-time is the time it would take a photon travelling at the speed of light to cross a distance equal to one Planck-length. Theoretically, this is the smallest time measurement that will ever be possible, roughly 10^{-43} seconds. Within the framework of the laws of physics as we understand them today, for times less than one Planck-time apart, we can neither measure nor detect any change."

Nothing occurs in that period. It is as if universe does not exist in that period. Nature just steps over this period. The steps need not be exactly equal to the Planck units, but they have the same order of magnitude. In the model these steps are taken in synchrony. This follows from the fact that a separable Hilbert space can only represent a static status quo. It also holds for a Hilbert space that is extended with static fields. In the Hilbert book model dynamics is implemented via universe wide progression steps. A progression step occurs when an extended Hilbert space is followed by a subsequent extended Hilbert space.

The origin of the existence of the space step follows from the inaccuracy of the coupling between the strand operator and the GPS operator.

The Hilbert book model uses the concept that the state of the universe can be considered as a sequence of static status quos. With respect to Einstein's special relativity this might at first sight seem an odd idea. This holds especially with respect to the relativity of simultaneity. However, as will be shown[16], special relativity perfectly fits the Hilbert book model.

The unit sphere of the Hilbert space is an affine space. It houses all unit length eigenvectors. This also holds for the eigenvectors of the position operator. This means that between two realizations of the Hilbert space the eigenvector that corresponds to the origin of position can be freely selected. Or with other words the origin of position can be selected freely.

Differences between positions in subsequent members of the sequence of extended separable Hilbert spaces can be interpreted as displacements. The displacement is a coordinate transformation. For the properties of this transformation it does not matter where the displacement starts or in which direction it is taken.

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[16] See "On the origin of physical dynamics; Dynamics; Relativity

The same holds for displacements that concern sequence members that are located further apart. The corresponding displacements form a group. The displacement is a function of both the position and the sequence number. The displacement $z, t \to z', t'$ can be interpreted as a coordinate transformation and can be described by a matrix.

$$\begin{bmatrix} t' \\ z' \end{bmatrix} = \begin{bmatrix} \gamma & \delta \\ \beta & \alpha \end{bmatrix} \begin{bmatrix} t \\ z \end{bmatrix} \tag{1}$$

The matrix elements are interrelated. When the displacement concerns a uniform movement, the interrelations of the matrix elements become a function of the speed v. The group properties together with the isomorphism of space fix the interrelations.

$$\begin{bmatrix} t' \\ z' \end{bmatrix} = 1/\sqrt{1 + kv^2} \begin{bmatrix} 1 & kv \\ -v & 1 \end{bmatrix} \begin{bmatrix} t \\ z \end{bmatrix} \tag{2}$$

If k is positive, then there may be transformations with $kv^2 \gg 1$ which transform time into a spatial coordinate and vice versa. This is considered to be unphysical. The Hilbert book model also supports that vision.

The condition k = 0 corresponds to a Galilean transformation

$$\begin{bmatrix} t' \\ z' \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -v & 1 \end{bmatrix} \begin{bmatrix} t \\ z \end{bmatrix} \tag{3}$$

The condition $k < 0$ corresponds to a Lorentz transformation. We can set $kc^2 = -1$, where c is an invariant speed that corresponds to the maximum of v.

$$\begin{bmatrix} t' \\ z' \end{bmatrix} = 1/\sqrt{1 - v^2/c^2} \begin{bmatrix} 1 & -v/c^2 \\ -v & 1 \end{bmatrix} \begin{bmatrix} t \\ z \end{bmatrix} \tag{4}$$

The Lorentz transformation corresponds with the situation in which a maximum speed occurs.

Since in each progression step photons step with a non-zero space step and both step sizes are fixed, the speed of the photon at microscopic scale is fixed. No other particle goes faster, so in the model a maximum speed occurs. With other words when sequence members at different sequence number are compared, then the corresponding displacements can be described by Lorentz transformations.

Lorentz transformations introduce the phenomena that go together with relativity, such as length contraction, time dilatation and relativity of simultaneity that occur when two inertial reference frames are considered.

$$\Delta t_c = \left(\Delta t_p - \Delta z_p \, v/c^2\right)/\sqrt{1 - v^2/c^2} \tag{5}$$

The term $\Delta z_p \, v/c^2$ introduces time dilatation. If $\Delta t_p = 0$ then depending on v and Δz_p the time difference Δt_c is non-zero.

These phenomena occur in the Hilbert book model when different members of the sequence of Hilbert spaces are compared. Usually the inertial frames are spread over a range of Hilbert book pages.

Since the members of the sequence represent static status quos, the relativity of simultaneity restricts the selection of the inertial frames. Only one of the inertial frames can be situated completely in a single member of the sequence. In that case the other must be taken from a range of sequence elements.

It means that when proper time is taken to be directly related with the progression parameter, thus when the corresponding inertial frame is fully located in a single sequence member, then coordinate time must differ from the progression parameter.

Continuity equations

All equations of motion are in fact continuity equations that treat the local information generation, annihilation and transfer.

Total change within V = flow into V + production inside V (1)

$$\frac{d}{dt}\int\limits_V \rho_0 \, dV = \oint_S \hat{\boldsymbol{n}}\rho_0 \frac{\boldsymbol{v}}{c} \, dS + \int\limits_V s_0 \, dV \tag{2}$$

$$\int\limits_V \nabla_0 \rho_0 \, dV = \int\limits_V \langle \boldsymbol{\nabla}, \boldsymbol{\rho} \rangle \, dV + \int\limits_V s_0 \, dV \tag{3}$$

Here $\hat{\boldsymbol{n}}$ is the normal vector pointing outward the surrounding surface S, $\boldsymbol{v}(q)$ is the velocity at which the charge density $\rho_0(q)$ enters volume V and s_0 is the source density inside V. $\boldsymbol{\rho}$ stands for $\rho_0 \boldsymbol{v}/c$.

The combination of c and $\boldsymbol{\rho}(q)$ is a quaternionic skew field $\rho(q)$ and can be seen as a probability amplitude distribution (PAD).

$$\rho = \rho_0 + \boldsymbol{\rho} \tag{3}$$

$\rho(q)\rho^*(q)$ can be seen as a probability density distribution (PDD).

Depending on their sign selection, quaternions come in four flavors. In a PAD the quaternion flavors do not mix. So, there are four PAD flavors.

Still these flavors can combine in pairs or in quadruples.

The field $\rho(q)$ contains information on the distribution $\rho_0(q)$ of the considered charge density as well as on the current density $\boldsymbol{\rho}(q)$, which represents the transport of this charge density.

Where $\rho(q)\rho^*(q)$ can be seen as a probability density of finding the center of charge at position q, the probability density distribution $\tilde{\rho}(p)\tilde{\rho}^*(p)$ can be seen as the probability density of finding the center of the corresponding wave package at location p. $\tilde{\rho}(p)$ is the Fourier transform of $\rho(q)$.

Lagrangian density

In physics the Lagrangian appears to be a very powerful instrument. With respect to the Hilbert book model (HBM) it appears to be not the proper entry point. A single Hilbert book page contains a complete description of the current static status quo. That means a complete description of the field configuration, which includes a description of the anchor points to the fields. These anchor points correspond to Hilbert vectors. When the fields are known, then also their Fourier transforms are known. This means that not only the probability distributions of positions are known, but also the probability distribution of momentums. Thus these data in fact comprise the complete description in terms of the Hamiltonian density rather than the description in terms of the Lagrangian density. Luckily enough the Hamiltonian density of the private field of each particle can be converted in a corresponding Lagrangian density, but curvature may hamper easy conversion. However, in general, locally the situation can be solved without much trouble. In this way the behaviour of a single private field in the environment constituted by all other private fields can be studied.

The consequence of this structure is that the Hamiltonian of a private field ψ of a free elementary particle does not explicitly contain the parameter t[17] and that this private field ψ becomes its time dependence by adding a phase:

$$\psi(q,t) = \chi(q)\exp(-iE\ t)$$

$$\tilde{\psi}(q,t) = \tilde{\chi}(q)\exp(-iE\ t)$$

$$H\psi = E\psi$$

$\tilde{\psi}(p)$ is the Fourier transform of $\psi(q)$. $\tilde{\chi}(p)$ is the Fourier transform of $\chi(q)$.

The Hamilton density $\mathcal{H}_{HBM}\big(\psi(q), \tilde{\psi}(p), t\big)$ in the Hilbert book model is then a function of $\psi(q)$, $\tilde{\psi}(p)$and t, while its representation $\mathcal{H}_{\mathrm{H}}\big(\chi(q), \tilde{\chi}(p)\big)$ in the Hilbert space **H** is a function of $\chi(q)$ and $\tilde{\chi}(p)$. This Hilbert space represent a single page of the book.

Information detection

All information that is transmitted by nature is carried by clouds of information carrying quanta (see figure 1).

[17] See: Eliahu Comay:, "Physical Consequences of Mathematical Principles", (Progress in Physics, October 2009 Vol 4), http://www.tau.ac.il/~elicomay/MathPhys.pdf

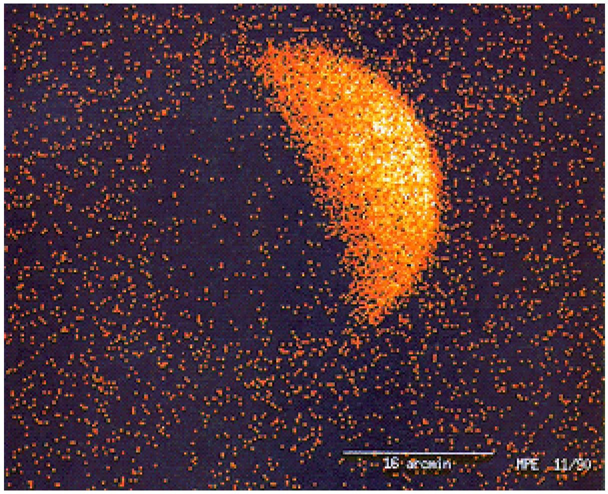

Figure 1: Intensified low dose image of the moon

The clouds themselves carry secondary information in their shape and their movement characteristics. It looks as if all quanta are generated by a series of Poisson processes. These facts become apparent when observations or measurements are done at very low dose rates [3]. The shape of the cloud is set by the corresponding PAD’s.

As indicated before, coherent states act as Poisson processes. The same holds for other PAD’s that support creation and annihilation of substates.

Rigged Hilbert space

The rigged partner **Ħ** of a separable Hilbert space **Ḩ** is not a separable Hilbert space, but a Gelfand triplet. It is an ordered set $(\Phi, Ḩ, \Phi^{x})$, where **Ḩ** is the Hilbert space used to generate Φ and Φ^{x}. The eigenspaces of normal operators in a Gelfand triplet need not be countable. They can be continuous spaces such as the full set of quaternions. The name of Hilbert is misused to identify the Gelfand triplet as a rigged Hilbert space. This paper uses the Gelfand triplet **Ħ** in order to provide a background GPS system and to couple the equivalent of the separate Hilbert space strand operator to the corresponding GPS operator. Both the equivalent strand operator and the GPS operator reside in the rigged Hilbert space **Ħ**. In this way the granules of the chains that reside in separable Hilbert space get their position. Another use of the background GPS operator is the coupling of field values to a position value. For that purpose the field values must be attached to the corresponding eigenvectors in rigged Hilbert space Ħ.

Discussion

The Hilbert book model

This model of physical reality does not contain singularities. Nor does it contain infinities. The only infinity it uses is the infinity of the dimension of the separable Hilbert space.

The model is fundamentally granular. The only continuities that the extended Hilbert space uses are the continuity of the background coordinate system that it borrows from its rigged partner and the continuity of the shapes of the PAD's.

Gravity and inertia

In the Hilbert book model, the gravitation field is treated as a derived field. It has long range effects due to the fact that its charges (the curvature values) do not get compensated by opposite charges as happens with electric charges. Prove is given by the existence of inertia, which can only be explained by analyzing the influence of the universe of particles on a local particle [3], [4]. Locally this influence causes an enormous potential Φ, which according to Sciama can be related to the gravitational constant G. Uniform movement of a particle does not raise other field activity than a field reconfiguration, but any acceleration of the particle goes together with an extra vector field [4].

Quantum clouds

The notion of quantum cloud needs clarification. The quantum cloud that corresponds to the private field of an elementary particle only contains the current granules of that particle as information carrying quanta. A field that consists of a superposition of the private fields of a set of elementary particles corresponds to a quantum cloud that contains quanta that correspond to the current granules of these particles. Not only the quanta carry information. Also the shape of the cloud that contains the quanta contains interpretable information.

When the cloud consists of emitted particles, then the process that controls the emission can be considered as a Poisson process. Upon detection an elementary particle is fully absorbed or it is converted into other particles from which at least one is absorbed. A detected particle was emitted by some body. During its travel it may have been reflected against or deflected by other bodies. The corresponding quantum clouds are affected correspondingly.

A quantum cloud can gain and lose quanta. An emission generates quanta and the corresponding private fields, which then form the shape of the cloud. The quantum cloud that corresponds to a private field disappears with its last quantum.

Testing theories

You can falsify a theory when its conclusions according to a selected logic are not consistent with its presumptions. If you take classical logic as a criterion then QM is a wrong theory. If you take quantum logic as a criterion, then more of quantum physics will pass, but you will have difficulty in checking quantum field theory. Only after extending quantum logic, such that it includes fields, you can handle quantum field theory as well. Still this equipment does only reach to test static situations.

Strand model

The main difference between the Hilbert space approach that is taken here and Schiller's approach lays in the interpretation of the source of the observable data. The principle fundamental postulate of Schiller's strand model is that the crossing switches of strands deliver the observable data. In the Hilbert book model the moving and rotating PAD's that are connected to the current granules carry the observable data.

Further, Schiller's strand model derives fields from strand tangles. In the Hilbert approach the shape and the fluctuation of the chains are controlled by fields.

In both pictures the described concepts may form the basis of a consistent model. Both models claim to deliver the proper equations of movement. [2], [3]. The reason of this conformance lays in the similarity of the basic field constituents.

Schiller's strand model takes its claims still further. He claims that this model fully explains the standard model and that no further particles than those specified by the standard model exist.

Apart from the difference with respect to the main postulate of strand model, an important difference exists between the approach presented in the Hilbert book model and Schiller's strand model. Schiller presents the gravitation field as a separate field that is mainly determined by distant fluctuations of tangle tails. The Hilbert book model treats the gravitation field as a derived field.

Summary of scratches

The following scratches have been treated here.

1. Due to its link with traditional quantum logic quantum a model of physics must be based on separable Hilbert spaces, but quite often it is based on a non-separable Hilbert space.
2. Neither the separable Hilbert space nor the rigged Hilbert space can represent dynamics. They can only represent a static status quo.
3. The separable Hilbert space cannot represent physical fields. It must be extended in order to cope with fields. In models based on a non-separable Hilbert space fields are often represented by operators.
4. Nature is fundamentally granular. The usual GPS-like operators do not support granularity.
5. It is impossible to represent a continuum GPS-like operator in separable Hilbert space.
6. Gravitation field is usually seen as an independent field.

Summary of repairs

The following repairs have been suggested.

1. Base quantum physics on a book of Hilbert spaces, where each page is an extended infinite dimensional separable Hilbert space that represents the current static status quo. The extension is done by blurring a subset of the Hilbert vectors.
2. Introduce a strand operator whose eigenspace consists of one dimensional chains of granules, where each granule gets its position from a background GPS coordinate system that is generated by a GPS operator that houses in rigged Hilbert space.
    a. One of the granules of each chain is special. It is blurred. A corresponding eigenvector gives it its position.
    b. The blur is a PAD. It anchors on the granule and on the corresponding Hilbert vector.

    c. The granule corresponds to a ground state of the PAD.
3. During dynamic steps the PAD keeps the chain smooth.
4. Elementary particles are anchored on the special granules of one or more chains. The corresponding PAD's together form the particle's private field, which is also its wave function.
5. Together the private fields form an overall covering field.
6. The static covering field can be decomposed into a rotation free longitudinal part and a divergence free transverse part.
    a. This decomposition runs along curved lines.
    b. The curvature can be used to define a derived curvature field.
    c. The private curvature field of a particle enables the determination of the mass of the particle.
7. Glancing through the pages of the book of Hilbert spaces reveals the dynamics of the system. Dynamics couples the static parts of the fields.

Open issues

Not treated here are field equations and the equations of motion. The equations of motion are intimately related to what happens during the infinitesimal progression steps that link the members of the sequence of extended Hilbert spaces.

The field equations are closely related to the dynamic equations that treat Hilbert fields and as can be expected show great resemblance with the Maxwell equations.

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